Enrollment No: ______________________Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name: Problem Solving-I

Subject Code: 5SC02PRS1 **Branch: M.Sc. (Mathematics)**

Semester: 2 **Date: 30/04/2019** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the following questions** **(07)**

a. Evaluate $\int_C (x^2 - y^2 + 2ixy)\, dz$, where C isthe contour $|z| = 1$. (01)

b. Show that $v_1 = (1, -2, -4)$ and $v_2 = (-3, 6, 12)$ forms a linearly dependent set in R^3. (01)

c. State Liouville's theorem. (01)

d. Find argument of the complex number $z = \frac{1+2i}{1-3i}$. (01)

e. The set $W = \{(x, y, z) |\ y = z = 1\}$ is subspace of R^3. (True/False) (01)

f. An analytic function with constant modulus is constant. (True/False) (01)

g. The function $f(z) = \frac{1}{z(z+1)}$ has singular point at $z = 1$. (True/False) (01)

Q-2 **Attempt all questions** **(14)**

a. Evaluate $\int_C \frac{\cos \pi z^2}{(z-1)(z-2)}\, dz$, where C isthe circle $|z| = 3$. (05)

b. Determine the analytic function $f(z) = u + iv$, if $v = \log(x^2 + y^2) + x - 2y$. (05)

c. Find the bilinear transformation which maps the points $z_1 = 1, z_2 = i$ and $z_3 = -1$ into the points $w_1 = i, w_2 = 0$ and $w_3 = -i$ respectively. (04)

OR

Q-2 **Attempt all questions** **(14)**

a. Determine the order of each pole and calculate residue at each of the pole of $f(z) = \frac{z^2}{(z-1)^2(z+2)}$. (05)

b. Prove that (05)

$$(i)\ \frac{(\cos 3\theta + i \sin 3\theta)^5 (\cos \theta - i \sin \theta)^3}{(\cos 5\theta + i \sin 5\theta)^7 (\cos 2\theta - i \sin 2\theta)^5} = \cos 13\theta - i \sin 13\theta,$$

$(ii)\ \sin^2 z + \cos^2 z = 1.$

c. Evaluate $\int_C |z|^2 dz$, where C is the boundary of the square whose vertices are at the points $(0,0),(1,0),(1,1)$ and $(0,1)$. (04)

Q-3 **Attempt all questions** **(14)**

a. Obtain the Laurent's series which represents the function $f(z) = \frac{1}{z^2-3z+2}$ for the regions $(i)|z| < 1$, $(ii)\ 1 < |z| < 2$, $(iii)|z| > 2$. (07)

b. Express the polynomial $p(x) = -9 - 7x - 15x^2$ as a linear combination of $p_1(x) = 2 + x + 4x^2, p_2(x) = 1 - x + 3x^2, p_3(x) = 3 + 2x + 5x^2$. (04)

c. Show that $W = \{(x,0,z): x \text{ and } z \text{ are real number}\}$ is subspace of R^3 with the standard operations. (03)

OR

Q-3 **Attempt all questions** **(14)**

a. Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. (07)

b. Prove that $\sinh^{-1} z = \log\left(z + \sqrt{z^2+1}\right)$. (04)

c. Verify the Cauchy-Riemann equations for the function $f(z) = z^3$. (03)

SECTION – II

Q-4 **Attempt the following questions** **(07)**

a. Find particular integral of $(4D^2 + 4D - 3)y = e^{2x}$. (01)

b. Express the quadratic form $2x^2 + 3y^2 + 6xy$ in matrix notation. (01)

c. Solve $(D-3)^3 y = 0$. (01)

d. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ satisfies the matrix equation $A^2 + kA - 2I = 0$, then what is the value of k? (01)

e. Find the Euclidean inner product of $u = (3,1,4,-5)$ and $v = (1,0,-2,-3)$. (01)

f. The degree of a differential equation $[1 + y'^2]^3 = y''^2$ is 6. (True/False) (01)

g. The function $f(x,y) = 4x^2 + y^2$ on $R: |x| \le 1, |y| \le 1$ satisfies Lipschitz condition. (True/False) (01)

Q-5 **Attempt all questions** **(14)**

a. Solve the following simultaneous equations (06)

$$\frac{dx}{dt} - 7x + y = 0, \frac{dy}{dt} - 2x - 5y = 0.$$

b. Find the rank of matrix by normal form, where $A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -1 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$. (05)

c. Find $\langle \boldsymbol{f}, \boldsymbol{g} \rangle$ if $\boldsymbol{f} = f(x) = 1 - x + x^2 + 5x^3, \boldsymbol{g} = g(x) = x - 3x^2$ and the inner product $\langle \boldsymbol{f}, \boldsymbol{g} \rangle = \int_{-1}^{1} f(x)g(x)\,dx$. (03)

OR

Q-5 **Attempt all questions** **(14)**

a. Apply method of variation of parameters to solve $(D^2 - 2D + 1)y = 3x^{\frac{3}{2}}e^x$. (06)

b. Solve $(D^3 - 6D^2 + 11D - 6)y = e^{-2x} + e^{-3x}$. (05)

c. Find the standard matrix for the linear transformation $T: R^3 \to R^4$ define by the formula $T(x,y,z) = (3x - 4y + z, x + y - z, y + z, x + 2y + 3z)$. (03)

Q-6 **Attempt all questions** **(14)**

a. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and verify Cayley-Hamilton theorem. (07)

b. Find particular integral of $(D^2 + 5D + 4)y = x^2 + 7x + 9$. (04)

c. Solve $\frac{dy}{dx} + \frac{y}{x} = x^3 - 3$. (03)

OR

Q-6 **Attempt all questions** **(14)**

a. Find the eigenvalues and the corresponding eigenfunctions of (07)

$$y'' + \lambda y = 0, y(0) = 0 = y'(1).$$

b. Determine whether the given vectors $v_1 = (2, -1, 3), v_2 = (4, 1, 2)$, $v_3 = (8, -1, 8)$ forms span for R^3. (04)

c. Let T be a transformation from R^2 into R^2 defined by $T(x_1, x_2) = (x_1 + 2x_2, 3x_1 - x_2)$. Then show that T is a linear transformation. (03)